Curl in Coordinate Systems

Consider now the curl of vector fields expressed using our coordinate systems.

Cartesian

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{\partial A_y(\bar{r})}{\partial z} - \frac{\partial A_z(\bar{r})}{\partial y}\right]\hat{a}_x + \left[\frac{\partial A_z(\bar{r})}{\partial x} - \frac{\partial A_x(\bar{r})}{\partial z}\right]\hat{a}_y + \left[\frac{\partial A_x(\bar{r})}{\partial y} - \frac{\partial A_y(\bar{r})}{\partial x}\right]\hat{a}_z$$

Cylindrical

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{1}{\rho}\frac{\partial A_z(\bar{r})}{\partial \phi} - \frac{\partial A_\phi(\bar{r})}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial A_\rho(\bar{r})}{\partial z} - \frac{\partial A_z(\bar{r})}{\partial \rho}\right]\hat{a}_\phi + \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\phi(\bar{r})\right) - \frac{1}{\rho}\frac{\partial A_\rho(\bar{r})}{\partial \phi}\right]\hat{a}_z$$

Spherical

$$\nabla \times \mathbf{A}(\bar{r}) = \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi(\bar{r})\right) - \frac{1}{r\sin\theta}\frac{\partial A_\theta(\bar{r})}{\partial\phi}\right]\hat{a}_r$$
$$+\left[\frac{1}{r\sin\theta}\frac{\partial A_r(\bar{r})}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}\left(r A_\phi(\bar{r})\right)\right]\hat{a}_\theta$$
$$+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r A_\theta(\bar{r})\right) - \frac{1}{r}\frac{\partial A_r(\bar{r})}{\partial\theta}\right]\hat{a}_\phi$$

Yikes! These expressions are **very** complex. Precision, organization, and patience are required to **correctly** evaluate the **curl** of a vector field !